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A DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODEL OF PAKISTAN’S ECONOMY

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MODEL OF PAKISTAN’S ECONOMY

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ABSTRACT

In this paper we study the dynamics of Pakistan’s economy from a theoretical perspective through the lenses of the New Keynesian theory. Various exogenous sources of business cycle fluctuations such as fiscal policy shock, cost push shock, oil price shock, risk premium shock and monetary policy shock have been internalized theoretically. The role of money to capture the impact of informal sector has been introduced via money in utility (MIU) function. Oil price shock allows us to understand the role of international commodity market prices and government spending is introduced as a major driving force that stimulates aggregate demand through crowding-in effect. Finally, risk premium shock is added to the uncovered interest parity (UIP) condition with the understanding that exchange rate fluctuations are mainly caused by factors other than interest rate differential due to incomplete and limited domestic financial markets. The study assumed that the State Bank of Pakistan follows the forward-looking Taylor-type monetary policy rule.

Key Words: New Keynesian Model, Forward-looking expectations, Micro-foundations.
1. INTRODUCTION

Modelling true structure of an economy to comprehend the contribution of various sources of business cycle fluctuations is an interesting field of research in modern macroeconomics. During the epoch of consolidation (1940-1980) fundamental changes have been witnessed in macroeconomic literature and ‘steady accumulation of knowledge’ became possible [Blanchard and Wolfers, (2000)]. Improvement in data collection techniques, more consistent econometric methods, detailed modelling of behavioral relationships, construction of aggregate macro-econometric models and incorporation of expectations are some of the notable achievements of this period. Keynes General Theory (1936) and Hicks IS-LM model (1937) laid the foundation of the Keynesian revolution which was extended in various directions by their followers. However, after the stagflation of 1970s these traditional macroeconomic models rooted in the Keynesian theory, failed to respond to new challenges, hence received heavy criticism by Sims (1980), Sargent and Wallace (1975), and others for their inapplicability to policy analysis and forecasting. The strongest criticism came from Lucas (1976) who questioned the policy relevance of the prevalent macro-econometric models. In fact, the continuous use of error learning models and absence of micro-foundations in Keynesian models rendered them almost ‘useless’ for policy analysis. Since the seminal contribution of Lucas (1976), the macroeconomic literature benefitted immensely from the contribution of Kydland and Prescott (1982) and Long and Plosser (1983) and many others.

The intellectual debate stressing the need for a forward-looking, micro-based macro model was rewarded in the shape of that are now regarded as the first generation of Dynamic Stochastic General Equilibrium (DSGE, hereafter) models. These frictionless models initially developed by extending the Ramsey-Case-Koopmans growth model with stochastic technology shock. The welfare optimization problem of forward-looking rational agents was solved subject to budget (household), technological (firms) and institutional constraints [Smets and Wouters (2007)]. Notwithstanding the powerful features, the RBCs came under serious criticism for their reliance on the assumption of classical dichotomy. Furthermore, absence of nominal and real rigidities made these models inappropriate for short-run policy (fiscal, monetary) analysis.
To overcome these intrinsic flaws, the New-Keynesians augmented their models by introducing nominal rigidities like, staggered price setting [Calvo (1983)], wage contracts [Fischer (1977) and Taylor (1980)], welfare analysis [Gali and Monacelli (2005), and Monacelli (2005)], exchange rate persistence [Chari et al., (2002)], exchange rate pass-through into domestic inflation [Devereux and Engel (2002)] and financial frictions [Del Negro et al., (2016), and Christiano et al., (2011)]. The inclusion of these features have significantly improved the existing DSGE models on theoretical as well as empirical grounds. Moreover, these modifications have enabled the DSGE models to adequately capture the short-run dynamics vis-à-vis monetary-cum-fiscal policy transmission mechanism [Christiano et al., (2005)].

To start with, the earlier DSGE models were developed for closed economy analysis. They were then extended to open economy while keeping intact the basic notions of rational behavior on the part of economic agents and various market frictions [GM, (2005); CGG, (2002); MN, (2000) among many others]. Today, the DSGE model has become the workhorse of quantitative analysis being used by many central banks for policy valuation (Tover, 2009).

Despite the promise shown by the DSGE models, it is surprising that limited progress has been made in developing economies in general and for Pakistan in particular. Naqvi. et al., (1983) can be regarded as the pioneering attempt within DSGE framework. Only few studies can be found e.g. Haider and Khan (2008), Choudhri and Malik (2012), Ahmed et al., (2012), Haider et al., (2012), Choudhri and Pasha (2013), Khan and Ahmed (2014), Ahmed and Pasha, (2014) and Nawaz and Ahmed (2015). Some of these studies are developed in a closed economy framework while others have extended the model to include open economy attributes. In general, Lubik and Schorfheide’s (2006) has been the base line model for some of these studies that also incorporate the informal sector.

In this paper we study the dynamics of Pakistan’s economy from a theoretical perspective through the lenses of the New Keynesian theory. Various exogenous sources of business cycle fluctuations such as fiscal policy shock, cost push shock, oil price shock, risk premium shock and monetary policy shock have been internalized theoretically. The role of money to capture the impact of informal sector has been introduced via money in utility
(MIU) function. Oil price shock allows us study to understand the role of international commodity market prices and government spending is introduced as a major driving force that stimulates aggregate demand through crowding-in effect. Finally, risk premium shock is added to the uncovered interest parity (UIP) condition with the understanding that exchange rate fluctuations are mainly caused by factors other than interest rate differential due to incomplete and limited domestic financial markets. The study assumed that the State Bank of Pakistan follows the forward-looking Taylor-type monetary policy rule.

The DSGE model presented here has three distinct features. First, it incorporates the role of money holding by its introduction in utility (MIU) function (see, Sevensson, 1985; Cooley and Hansen, 1997; 1998). Second, to understand business cycle fluctuations volatility is introduced through imported oil which has witnessed fluctuations over the past few decades. Since, import of petroleum products is the largest component of Pakistani import bill since long, oil is incorporated in the production function as basic input. Third, keeping in view the role of fiscal policy shock, government spending is added to aggregate demand function. The role of money demand is inevitable in case of Pakistan for various reasons. (a) Different financial access indicators\(^\text{1}\) demonstrate that financial deepening has yet to take roots in Pakistan. Lack of financial deepening pertains to limited provision of financial services to the society that stimulates individuals to hold more money and involves higher level of currency in circulation; (b) the existence and operation of large scale informal (undocumented) sector is directly related to the level of currency in circulation\(^\text{2}\). High levels of currency in circulation facilitate economic agents to make transactions in cash instead of appropriate financial channels and left undocumented; and (c) as Ahmed and Pasha, (2014) found that monetary aggregates for Pakistan are positively associated with current GDP, and hence, there is ample justification to model the role of money for Pakistan theoretically. The rest of this paper includes derivation of the model. Schematic diagram as under.

\(^{1}\) Deposit accounts per capita, number of bank branches per million adults and loan accounts per thousand adults.

\(^{2}\) While tax on cash withdrawal was aimed to reduce the size of cash transactions to reduce informal (undocumented) sector, this effort has been severely and adversely impacted by tax on financial and banking sector transactions.
2. THE SMALL OPEN ECONOMY MODEL

In this section we describe the salient features of an SOE- DSGE model which is derived to represent the structure and functioning of the economy of Pakistan. This forward looking model has micro-foundations and it is closely linked to the SOE literature of Gali and Monacelli (2005) (GM, 2005 henceforth), Clarida et al, (CGG henceforth) (1999; 2002) and McCallum and Nelson (MN henceforth) (1999, 2000). To facilitate the understanding of the structure of the model and the process of by decision making economic agent, we make use of a schematic diagram as under.

The basic concept is taken without any alteration from GM (2005). The world economy is assumed to be a continuum of small economies. The domestic economy is one among those economies, relatively very small in size (is of measure zero)\(^3\) with respect to the size of the rest of the world’s economies. Hence, the actions (decisions) of domestic

---

\(^3\) See, Gali and Monacelli, 2005.
economic agents do not influence variables related to rest of the world like, consumer price level, interest rate, oil price and aggregate demand. These economies are interconnected and share similar preferences and technology. Economic agents are rational and forward-looking who make best use of all available information. Domestic prices and wages are sticky and adjust infrequently and are partially indexed to expected inflation. This is the most powerful assumption, which enhances the realism of the model.

We have introduced certain modifications to the original model to keep it tractable so that it represents the real mechanics of Pakistan’s economy and serve the required task of policy analysis. First, oil is introduced into the production process as basic input where oil prices are assumed to be exogenously determined. Secondly, in view of the existence of large scale public sector in Pakistan. Hence, the role of fiscal policy is assessed by adding government spending to the aggregate demand function. Like other developing economies, labour markets in Pakistan confront various types of rigidities. Hence, we have assumed them to be imperfect due to the presence of high rate of unemployment and minimum wage laws that distort equilibrium wages and the demand for labour. Finally, due to imperfect domestic financial markets and limited access to international financial markets along with huge national and international debt, risk premium is introduced into the uncovered interest parity condition to hold in the short-run.

2.1 THE DEMAND SIDE

2.1.1 Households Problem

The household problem is standard in nature where representative domestic household is infinitely lived. The household’s consumption basket include domestically produced goods and foreign goods imported from rest of the world. Household also provide differentiated labour effort to firms and get the equivalent wage compensation. Furthermore, firms are owned by the household who receive profits generated by these monopolistically competitive firms.

The preferences of the representative household seeking to maximize life time utility is defined through the following separable utility function,

\[ U = U \left( C_t, N_t, \frac{M_t}{P_t} \right) = E_0 \sum_{t=0}^{\infty} \beta^t \left( U(C_t) - V(N_t) + W \left( \frac{M_t}{P_t} \right) \right) \]
Where \( U(C_t, N_t, M_t/P_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \frac{(M_t/P_t)^{1-\theta}}{1-\theta} \)

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \frac{(M_t/P_t)^{1-\theta}}{1-\theta} \right)
\]

(1)

So that the period utility function becomes where \( N_t, C_t, M_t/P_t \) denotes labour effort, aggregate consumption index and demand for real cash balances. Furthermore, \( 0 < \beta < 1 \) is the discount factor, \( \sigma \) represents the degree of relative risk aversion and \( \frac{1}{\sigma} \) is the elasticity of inter-temporal substitution. \( \theta \) is the inverse of the interest elasticity of real money holding and \( \phi \) stands for the inverse of the elasticity of labour supply. Marginal utility functions with respect to each variable are given as,

\[
U_{c,t} = \frac{\partial U(C_t, N_t, M_t/P_t)}{\partial C_t} > 0, \quad U_{N,t} = \frac{\partial U(C_t, N_t, M_t/P_t)}{\partial N_t} < 0, \quad U_{M,t} = \frac{\partial U(C_t, N_t, M_t/P_t)}{\partial (M_t/P_t)} > 0
\]

where, \( U_{c,t}, U_{N,t} \) and \( U_{M,t} \) stands for marginal change in utility due to consumption, labour effort and money holding. The signs attached-with, implies current utility is positively associated with consumption and money holding but negatively associated with labour effort.

\[
U_{cc,t} = \frac{\partial^2 U(C_t, N_t)}{\partial c_t^2} < 0, \quad U_{NN,t} = \frac{\partial^2 U(C_t, N_t)}{\partial N_t^2} > 0, \quad U_{MM,t} = \frac{\partial^2 U(C_t, N_t)}{\partial M_t^2} < 0
\]

The above relations show the rate of change of utility level in response to the concerned variables, implies that the contribution to utility by consumption, money holding and labour effort diminishes. To simplify the analysis, we further assume that \( U_{MN,t} = U_{CM,t} = U_{CN,t} = 0 \) indicates marginal utility of any specific element is independent of the level of other elements.

The household seeks to maximize its lifetime utility. The representative household demand for differentiated goods having different degrees of substitutability which furnishes the profit maximizing firms with market power and enable them to generate monopolistic rent. The decision problem vis-à-vis utility maximization can be decomposed into two stages, initially, for any given level of income (consumption expenditure) the household
seeks for an optimal consumption bundle that maximizes the total consumption. In the second stage, given the optimal consumption bundle the household decides over the optimal combination of consumption, labour and money holding to maximize life-time utility.

Each consumer demands for a consumption bundle \( C_t \) that consists of differentiated goods \( c_{jt} \) produced either by domestic or foreign firms.

\[
C_t = \left[ (1 - \alpha)^{\gamma} C_{H,t}^{(y-1)/y} + \alpha^{\gamma} C_{F,t}^{(y-1)/y} \right]^{y-1/y} \tag{2}
\]

where, \( C_{H,t} \) and \( C_{F,t} \) represents an index of consumption of domestically produced goods and imported goods defined by CES function,

\[
C_{H,t} = \left[ \int_0^1 C_{H,t} (j)^{(\theta-1)/\theta} dj \right]^{\theta-1/\theta} \quad \text{and} \quad C_{F,t} = \left[ \int_0^1 C_{F,t} (j)^{(y-1)/y} dk \right]^{\gamma/y}
\]

where, \( C_{k,t} \) is an index of foreign goods produced by country k and consumed by the domestic consumer defined as:

\[
C_{k,t} = \left[ \int_0^1 C_{k,t} (j)^{(\theta-1)/\theta} dj \right]^{\theta/\theta-1}
\]

where, \( \theta > 1 \) is the elasticity of substitution among goods produced within any country, \( \gamma > 0 \) shows the substitutability between domestic and foreign goods. \( 0 < \alpha < 1 \) measures the share of expenditures on foreign goods in the consumption basket of domestic household and is generally considered as degree of openness in case of a small open economy.

Utility maximization defined by (1) is subject to the following lifetime budget constraint:

\[
\int_0^1 P_{H,t} (j) C_{H,t} (j) dj + \int_0^1 \int_0^1 P_{k,t} (j) C_{k,t} (j) dk dJ + E_t \{ F_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t \tag{3}
\]

Where \( P_{H,t} (j) \) the price of is domestically produced well and \( P_{k,t} (j) \) is the price of foreign good imported from country k in terms of domestic country’s currency. \( D_{t+1} \) is the nominal return received by the household in period \( t+1 \) for the portfolio held in period \( t \) and \( F_{t,t+1} \) is the stochastic discount factor. The portfolio includes shares in firms and console (one period
bonds) issued by either domestic or foreign government. \( W_t \) is the nominal wage rate and \( T_t \) are the lump sum transfers.

The demand function for each variety of good for a given expenditure are as follow:\(^4\):

\[
C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\gamma} C_{H,t} \quad ; \quad C_{k,t}(j) = \left( \frac{P_{k,t}(j)}{p_{k,t}} \right)^{-\gamma} C_{k,t} 
\]

where, \( P_{H,t} = \left[ \int_0^1 P_{H,t}(j)^{1-\theta} dj \right]^{1-\theta} \) is the price index for domestically produced goods and \( P_{k,t} = \left[ \int_0^1 P_{k,t}(j)^{1-\gamma} dj \right]^{1-\gamma} \) is the price index for goods imported from country \( k \) denominated in domestic country’s currency further it implies that: \( P_{H,t}C_{H,t} = \int_0^1 P_{H,t}(j)C_{H,t}(j) dj \) and \( P_{k,t}C_{k,t} = \int_0^1 P_{k,t}(j)C_{k,t}(j) dj \). Where the total expenditure on foreign goods imported from rest of the world can be written as \( P_{F,t}C_{F,t} = \int_0^1 P_{F,t}C_{F,t} dk \).

Conditional on expenditure minimization and optimal allocation of expenditure between domestic and foreign goods can be expressed as:

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{p_t} \right)^{-\gamma} C_t \quad ; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{p_t} \right)^{-\gamma} C_t 
\]

(4)

Finally, the consumer price index in the home country may be defined as:

\[
P_t = \left[ (1 - \alpha)P_{H,t}^{1-\gamma} + \alpha P_{F,t}^{1-\gamma} \right]^{1/(1-\gamma)}
\]

(5)

2.1.2 Utility Maximization

Since the utility function is nested function of \( c_{jt} \), it implies that an increase in consumption will increase the life time utility. The total consumption expenditure of domestic household may be defined as:

\[
P_tC_t = P_{F,t}C_{F,t} + P_{H,t}C_{H,t}
\]

Thus, the budget constraint (3) can be written as:

\(^4\) See Appendix (A) for detailed derivation.
\[ P_t C_t + E_t \{ F_{t,t+1}D_{t+1} \} + M_t \leq D_t + W_t N_t + M_{t-1} + T_t \] (6)

The behavior of the representative optimizing agent facing the above budget constraint can be described by the intertemporal optimality condition:

\[ C_t^\sigma N_t^{\varphi} = \frac{W_t}{P_t} \] (7)

Euler equation which shows the inter-temporal substitution in consumption

\[ \beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1 \] (8)

and the money demand function is

\[ \frac{M_t}{P_t} = C_t^{\frac{\sigma}{\varphi}} \left( \frac{1+i_t}{i_t} \right)^{\frac{1}{\varphi}} \] (9)

Log-linearized form of equations (7, 8 and 9) are:

\[ w_t - p_t = \sigma c_t + \varphi n_t \] (10)

\[ c_t = E_t \{ c_{t+1} \} - \frac{1}{\sigma} (r_t - E_t \{ \pi_{t+1} \} - \rho) \] (11)

\[ m_t - p_t = \left[ \frac{\sigma}{\varphi} \right] c_t - \frac{1}{\sigma} \left[ \frac{1}{(1+i_t)} \right] i_t \] (12)

where, \( w_t = \log W_t, \ p_t = \log P_t, \ c_t = \log C_t \) and \( n_t = \log N_t, \ m_t = \log M_t \) \( p_t = \log P_t \) and \( E_t \{ \pi_{t+1} \} = E_t p_{t+1} - p_t \). Equation (10) implies that supply of labour is a function of existing and reservation wage differential. If the existing real wage is found to be higher than the reservation wage, the workers make more effort and vice versa (Lucas and Rapping, 1969). Equation (11) describes the consumption smoothing behavior. In general, given the diminishing marginal utility in consumption, the forward-looking household tries to smooth her consumption pattern through lending and borrowing. The first term implies a positive relationship between expected consumption and current consumption, and the second term shows an inverse relationship between current consumption and real interest rate. The last

\^ See Appendix-B for detailed derivation
equation (12) is the money demand function which shows that money demand is positively associated with income (current consumption) and decreases due to an increase in interest rate. The nominal interest rate is \( log_i_t = log(1 + i_t) \approx i_t \) and \( \rho = -log\beta \).

### 2.2 Allocation of Government Spending

Like many other studies including Corsetti and Pesenti (2005), Ganelli (2005) and Perotti (2005) to simplify the analysis, we assume that government spending is absolutely home-biased, i.e., Government allocates funds to the purchase of domestically produced goods only. Government faces the same cost minimization problem subject to market price index. Thus, the government spending minimization problem can be expressed as:

\[
\min \int_0^1 p_{j,t} G_{j,t} \quad \text{Subject to} \quad G_t = \left[ \int_0^1 G_t(j)(\theta^{-1})/\theta \, dj \right]^{\theta-1} \\
G_t = \left( \frac{p_{H,t}(j)}{p_{H,t}} \right)^{-\theta} G_t
\]

Moreover the government makes transfer payments to the household and pays interest on government bonds. We further assume that the government runs a balanced budget in the long-run and finances its expenditures through lump-sum taxes, deficit financing and issuing one period bond. The transversality condition is imposed which implies that the government obeys its budget constraint. Finally the government will retire the debt. Furthermore, tax receipts and seigniorage revenues are either spent on purchase of goods and services or returned to households as transfer payments (McCallum and Nelson, 1999).

\[
P_{H,t}G_t + i_t B_t = T_t + (B_{t+1} - B_t) + \frac{M_t - M_{t-1}}{p_t} \\
\]

or

\[
P_{H,t}G_t = T_t + B_{t+1} - (1 + i_t)B_t + \frac{M_t - M_{t-1}}{p_t} \\
\]

Transversality condition implies: \( B_{t+1} - (1 + i_t)B_t = 0 \)
Therefore, we left with:

\[ P_{H,t}G_t = T_t + \frac{M_t - M_{t-1}}{P_t} \]  

(14)

2.3 CPI AND DOMESTIC INFLATION, REAL EFFECTIVE EXCHANGE RATE AND TERMS OF TRADE

The bilateral terms of trade \( S_{i,t} \) refers to the price of country i’s good in terms of domestic good. It can be defined as \( S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \) and the effective terms of trade therefore would be \( S_t = \frac{P_{F,t}}{P_{H,t}} \) whose log-linearizing rate yields, \( s_t = p_{F,t} - p_{H,t} \).

Following (GM, 2005) the CPI around symmetric steady state can be written as:

\[ p_t = (P_{H,t})^{1-\alpha} (P_{F,t})^\alpha \] whose log-linearization yields:

\[ p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} \]  

(15)

Solving for \( p_{F,t} \) and putting value in the definition of effective terms of trade results in:

\[ p_t = p_{H,t} + \alpha s_t \]  

(15a)

Forwarding one period ahead gives

\[ p_{t+1} = p_{H,t+1} + \alpha s_{t+1} \]  

(15b)

Now subtracting original equation (15a) from the forwarded equation (15b) we get:

\[ \pi_{t+1} = \pi_{H,t+1} + \alpha \Delta s_{t+1} \]

It is clear from the above equation that the difference between CPI and domestic inflation depends on the degree of openness (\( \alpha \)) and terms of trade variation, implying that the larger is the value of \( \alpha \) the more vulnerable the domestic economy to foreign inflation will be.
Now defining the bilateral real exchange rate \( Q_{i,t} \) as: 
\[
Q_{i,t} = \frac{V_{i,t}P_t^i}{P_t}
\]
where \( V_{i,t} \) is the nominal exchange rate and \( P_t^i \) is the country i’s CPI, \( P_t \) is the domestic CPI and \( P_t^F \) is the aggregate rest of the world’s CPI. Assuming that the law of one price holds. The real exchange rate (REER henceforth) can be defined as: 
\[
Q_t = \frac{P_{F,t}}{P_t}
\]
whose log-linearizing yields: 
\[
q_t = p_{F,t} - p_t.
\]

Using the log-linearized form of domestic CPI round symmetric steady state: 
\[
p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t},
\]
we redefine the relationship between CPI, domestic inflation, REER as below.

\[
p_t = p_{H,t} + \frac{\alpha}{1-\alpha}q_t
\]

(16)

\[
\pi_{t+1} = \pi_{H,t+1} + \frac{\alpha}{1-\alpha}\Delta q_{t+1}
\]

(17)

2.4 FINANCIAL MARKETS

2.4.1 International Risk Sharing Condition

Under the assumption of complete assets markets and similar preference across different economies, the household whether living in the small open economy or rest of the world, face exactly the same optimization problem. Hence \( \beta = \beta^* \), \( R_t = R_t^* \), \( C_t = C_t^* \) and \( P_t^* = P_{F,t}^* \). This means that the intertemporal optimality conditions (the Euler equation) holds exactly for all the households (domestic and foreigners). We can, therefore express the foreign economy version of the Euler equation (denominated in domestic currency) as:

\[
\beta^*R_t^*E_t\left\{\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{V_{t+t}^*}{V_{t+1}^*P_{t+1}^*}\right)\right\} = 1
\]

Equating the Euler equations for domestic and foreign household we get:

\[
\beta^*R_t^*E_t\left\{\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{V_{t+t}^*}{V_{t+1}^*P_{t+1}^*}\right)\right\} = \beta R_t E_t \left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right)\right\}
\]
\[ E_t \left\{ \frac{C_t}{C_{t+1}} \left( \frac{v_{t+1}^*}{v_{t+1}^*} \right) \right\} = E_t \left\{ \frac{C_t}{C_{t+1}} \left( \frac{p_t}{p_{t+1}} \right) \right\} \]

\[ E_t \left\{ (C_t)^\sigma \left( \frac{p_t}{p_{t+1}} \right) \right\} = E_t \left\{ (\frac{C_{t+1}}{C_{t+1}})^\sigma \left( \frac{v_{t+1}^*}{v_{t+1}^*} \right) \right\} C_t^\sigma \]

\[ C_t = E_t \left\{ \left( \frac{p_{t+1}}{v_{t+1}^*} \right)^{1/\sigma} \left( \frac{C_{t+1}}{C_{t+1}^*} \right) \right\} \left( \frac{v_{t+1}^*}{p_t} \right)^{1/\sigma} C_t^* \]

Since expression \( E_t \left\{ \left( \frac{p_{t+1}}{v_{t+1}^*} \right)^{1/\sigma} \left( \frac{C_{t+1}}{C_{t+1}^*} \right) \right\} = 1 \), i.e., the expression is constant and equal to one under symmetric steady state with the same initial endowments or zero net foreign assets holdings (see GM, 2005). Hence, \( C_t = C_t^* Q_{t,t}^{1/\sigma} \) whose log-linear version, reduces to:

\[ c_t = c_t^* + \frac{1}{\sigma} q_t \quad (18) \]

Which is known as the risk sharing condition. It implies that domestic consumption is a function of international consumption instead of domestic economy’s own current, lagged or lead income due to complete financial markets. However, even if the financial markets are incomplete the consumption growth across countries will be the same so long as the PPP holds (Heaton & Lucas, 1993 and 1995). Therefore, we don’t need to relax the assumption of perfect capital markets up to this point.

2.4.2 Uncovered Interest Parity Condition

Under the assumption of complete financial markets, the domestic investor can invest both in domestic and foreign bonds to minimize the idiosyncratic risk. Thus in the presence of foreign bonds, the budget constraint faced by the domestic consumer will be:

\[ P_t C_t + F_{t,t+1} B_{t+1} + F_{t,t+1}^* v_{t+1} B_{t+1}^* \leq B_t + v_t B_t^* + W_t N_t + T_t \]

The revised optimality conditions of the domestic household:
\[ \beta E_t \left( \frac{(C_{t+1}^C)}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) / F_{t,t+1} = 1 \] 

(19.a)

\[ \beta E_t \left( \frac{(C_{t+1}^C)}{C_t} \right)^{-\sigma} \left( \frac{V_{t+1}^t}{V_t} \right) / F_{t,t+1}^* = 1 \] 

(19.b)

Dividing (19.a) by (19.b)

\[ 1 = \frac{\beta E_t \left( (C_{t+1}^C)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) / F_{t,t+1} \right)}{\beta E_t \left( (C_{t+1}^C)^{-\sigma} \left( \frac{V_{t+1}^t}{V_t} \right) / F_{t,t+1}^* \right)} = \frac{E_t \left( F_{t,t+1}^V \right)}{E_t \left( F_{t,t+1}^V \right)} \frac{E_t \left( V_{t+1}^t \right)}{E_t \left( V_t \right)} = \frac{F_{t,t+1}^V}{F_{t,t+1}} \frac{V_{t+1}^t}{V_t} = \frac{i_t}{i_t^*} \]

Log-linearizing yields

\[ i_t - i_t^* = E_t \sigma_{t+1} - \sigma_t \]

\[ \Delta E_t \{ \sigma_{t+1} \} = i_t - i_t^* \]

But \( q_t = p_{F,t} - p_t \) and \( p_{F,t} = \sigma_t + p_t^* \), hence \( q_t = \sigma_t + p_t^* - p_t \)

\[ \Rightarrow \sigma_t = q_t + p_t - p_t^* \]

Forwarding one period ahead we get;

\[ \sigma_{t+1} = q_{t+1} + p_{t+1} - p_{t+1}^* \]

Substituting in UIP condition:

\[ i_t - i_t^* = +E_t q_{t+1} + E_t p_{t+1} - E_t p_{t+1}^* - q_t - p_t + p_t^* \]

Rearranging;

\[ E_t \{ \Delta q_{t+1} \} = (i_t - E_t \{ \pi_{t+1}^* \}) - (i_t^* - E_t \{ \pi_{t+1}^* \}) \]

This is the well-known UIP condition under perfect capital markets.

We now relax the assumption of perfect capital markets, and assume that uncovered interest parity condition does not hold and risk premium is involved for the foreign investor to invest in domestic bonds due to default risk. In this case the UIP condition holds in the following form:
\[ \Delta E_t \{v_{t+1}\} = (i_t - i_t^*) - \varepsilon^{\sigma^*} \]

\[ E_t \{\Delta q_{t+1}\} = (i_t - E_t(\pi_{t+1})) - (i_t^* - E_t(\pi_{t+1}^*)) - \varepsilon^{\sigma^*} \]  \hspace{1cm} (20)

Where \( \varepsilon^{\sigma^*} \) is the risk premium paid to foreign investor to compensate for holding domestic risky bond. We further assume that risk premium is positively related to the default risk in case of over-borrowing. The risk sharing condition implies that the domestic economy can be shielded against internal idiosyncratic shocks but at relatively higher cost. Moreover the cost associated with risk premia is nested with the risk of default.

2.5 SUPPLY SIDE

2.5.1 Firm Problem

The firm’s problem consists of two steps and is solved in two stages. In the first stage firm decides over the least cost combination of inputs subject to existing technology and its specific demand function, and in the second stage firms seeks to maximize its profits depending upon the revenue generated from selling the product at optimal price conditioned that prices are sticky.

2.5.2 Cost Minimization Problem

Assuming that oil and labour are the only inputs needed to produce the specific good. The firm tries to minimize its cost of production by choosing the least cost combination of the two inputs (oil and number of workers) subject to given prices of these inputs and prevailing state of technology. The short-run production function can therefore be written as:

\[ Y_t(j) = [A_t N_t(j)]^\eta [O_t^d(j)]^{1-\eta} \]  \hspace{1cm} (21)

where, \( O_t^d(j) \) is the amount of oil demanded by firm \( j \) as an intermediate input to produce one unit of output, \( \eta \) is the share of labour in total output and \( (1 - \eta) \) is the share of oil. For simplicity we assume perfect competition in labour market, (later on this assumption will be relaxed). Log of productivity \( a_t = \log(A_t) \) is assumed to be stochastic and follows an AR(1) process \( a_t = \rho a_{t-1} + \varepsilon_t^a \). Where \( \{\varepsilon_t^a\} \) is an i.i.d. independently and identically
distributed) shock to price of oil and \( \rho_o \in [0,1] \), \( P_{o,t} \) is the price of oil which is exogenously determined.

\[
p_{o,t} = \rho_o p_{o,t-1} + \varepsilon_t^o
\]  

(22)

Solving the first order conditions yield:

\[
(1 - \eta) W_t N_t(j) = \eta o_t^d(j) P_{o,t}
\]  

(23)

The nominal marginal cost of the firm is:

\[
MC_t^n = \frac{W_t}{\eta A_t^\eta N_t(j)^{\eta-1} a_t^\eta(j)^{1-\eta}}
\]

Using cost minimization condition the \( MC_t^n \) can be written as:

\[
MC_t^n = \frac{W_t \rho_{o,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{(1-\eta)A_t^\eta}}
\]

The real marginal cost in terms of domestic price can be expressed as:

\[
MC_t^n = \frac{W_t \rho_{o,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{(1-\eta)A_t^\eta} P_{H,t}}
\]

Log-linearizing the real marginal cost expression yield,

\[
mc_t^r = \eta w_t + (1 - \eta) p_{o,t} - \eta a_t - p_{H,t}
\]  

(24)

It implies that the marginal cost function is increasing in price of oil and nominal wage, but decreasing in productivity growth. The demand for oil by firm \( j \) is given by:

\[
o_t^d(j) = \left[ \frac{\eta(1+\varphi)}{1+\varphi(1-\eta)} \right] y_t - \left[ \frac{\eta(1+\varphi)}{1+\varphi(1-\eta)} \right] a_t - \left[ \frac{\eta}{1+\varphi(1-\eta)} \right] p_{o,t}
\]  

(25)

The aggregate domestic output over all firms is \( Y_t = \int_0^1 Y_t(j)^{(\theta-1)/\theta} d_j \theta^{-1} \). Lastly, the log-linearized version of the production function is

\[
y_t = \eta a_t + \eta m_t + (1 - \eta) o_t^d
\]  

(26)
2.5.3 Firm Profit Maximization

In the second stage, given that prices are sticky, firms choose an optimal price that maximizes their profits. Following Calvo (1983) firms are assumed to share identical technology, face monopolistic competition and produce differentiated goods. Due to price stickiness a fraction \( \varphi \) of firms is incapable of adjusting its price in period \( t \) and stick to the price that prevailed in period \( t-1 \). Thus \( \varphi \) is naturally an index of price stickiness and represents the probability that firm \( j \) will not be able to adjust its price in period \( t \). Then the firm’s profit maximizing pricing strategy yield the following Phillips curve\(^6\);

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \bar{m}c_t
\]

(27)

Where \( \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \) and \( \varphi_{t+1} = \bar{m}c_t \) is the log deviation of real \( mc_t \) from its flexible price equilibrium. Now following CGG, (1999, 2002) we relax the assumption of perfect labour markets and introduce a cost push shock to the Phillips curve, so that equation (7) can be written as:

\[
C_t^\sigma N_t^\varphi exp^{\ell}w = \frac{\bar{w}_i}{p_t}
\]

(28)

where, \( exp^{\ell}w \) is the wage markup, reflecting government intervention in terms of minimum wage laws in the labour market that would distort real wage from its equilibrium level under perfect markets. This allows us to rewrite equation (27) as follows:

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \bar{m}c_t + \mathcal{E}_t^w
\]

(29)

2.6 DEMAND SIDE EQUILIBRIUM

Goods market equilibrium requires

\[
Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,k,t}(j) dk + G_t
\]

(30)

---

\(^6\) For detailed derivation see Appendix-II to GM, 2005.
Where \( Y_t(j) \) is the total production of good \( j \) by all domestic firms, \( C_{H,t}(j) \) is the total consumption demand by domestic household for domestically produced goods \( j \) and \( \int_0^1 C_{H,t}^k(j) dk \) is country \( k \)'s demand for good \( j \) (exports) or imports.

\[
Y_t(j) = (1 - \alpha) \left( \frac{P_{H,t}(j)}{P_t} \right)^{-\theta} C_t + \alpha \left( \frac{P_{H,t}(j)}{P_t} \right)^{-\theta} \int_0^1 \left( \frac{P_{H,t}}{\gamma_{k,t} P_{F,t}} \right)^{-\theta} \left( \frac{P_{F,t}}{P_{H,t}} \right)^{-\gamma} C_t^k + \left( \frac{P_{H,t}(j)}{P_t} \right)^{-\gamma} G_t
\]

\[
Y_t(j) = \left( \frac{P_{H,t}(j)}{P_t} \right)^{-\theta} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\gamma_{k,t} P_{F,t}} \right)^{-\theta} \left( \frac{P_{F,t}}{P_t} \right)^{-\gamma} C_t^k + \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} G_t \right]
\]

But

\[
Y_t(j) = \left( \frac{P_{H,t}(j)}{P_t} \right)^{-\theta} Y_t \quad \text{or} \quad Y_t = Y_t(j) \left( \frac{P_{H,t}(j)}{P_t} \right)^{\theta}
\]

Using the optimal allocation of resources for SOE and ROW, REER definition and the behavioral similarity assumption we get,

\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left[ \left( 1 - \alpha \right) \int_0^1 (S_t^k S_{k,t})^{\gamma - \theta} Q_{k,t}^{\gamma - \frac{1}{\sigma}} dk \right] C_t + G_t
\]

where, \( \int_0^1 (S_t^k S_{k,t})^{\gamma - \theta} dk = 1 \) in the symmetric steady state hence,

\[
Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left[ \left( 1 - \alpha \right) \int_0^1 Q_{k,t}^{\gamma - \frac{1}{\sigma}} dk \right] C_t + G_t
\]

Log-linearizing around symmetric steady state yields,

\[
y_t = c_t + \gamma \left( p_t - p_{H,t} \right) + \alpha \left( \gamma - \frac{1}{\sigma} \right) q_t + g_t \quad (31)
\]

Rearranging and using equation \( \text{eq. 16} \) \( p_t - p_{H,t} = \frac{\alpha}{1-\alpha} q_t \) we can re-write \( \text{eq. 31} \) as below:

\[
y_t = c_t + \alpha \left( \frac{(2-\alpha)\gamma}{1-\alpha} - \frac{1}{\sigma} \right) q_t + g_t \quad (32)
\]
Assuming $c_t = y_t^*$ substituting in risk sharing condition $c_t = y_t^* + \frac{1}{\sigma} q_t$ and finally substituting in equation (32) we get,

$$y_t = y_t^* + \left[ \frac{\alpha \sigma (2 - \alpha) + (1 - \alpha)^2}{\sigma (1 - \alpha)} \right] q_t + g_t$$

Now the Euler equation can be written as (ignoring constant term)

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\})$$

Substituting in equation (32) we get

$$y_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{H,t+1}\}) - \left[ \frac{\alpha (2 - \alpha)(\gamma \sigma - 1)}{\sigma (1 - \alpha)} \right] q_t + g_t$$

Forwarding equation (32) one period ahead.

$$E_t\{y_{t+1}\} = E_t\{c_{t+1}\} + \alpha \left( \frac{(2 - \alpha)\gamma}{1 - \alpha} - \frac{1}{\sigma} \right) E_t\{q_{t+1}\} + E_t g_{t+1}$$

Solving for $E_t\{c_{t+1}\}$;

$$E_t\{c_{t+1}\} = E_t\{y_{t+1}\} - \alpha \left( \frac{(2 - \alpha)\gamma}{1 - \alpha} - \frac{1}{\sigma} \right) E_t\{q_{t+1}\} - E_t g_{t+1}$$

and substituting in equation (34), after rearranging we get:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{H,t+1}\}) - \left[ \frac{\alpha (2 - \alpha)(\gamma \sigma - 1)}{\sigma (1 - \alpha)} \right] E_t\{\Delta q_{t+1}\} - E_t\{\Delta g_{t+1}\}$$

But from equation (17):

$$E_t\pi_{t+1} = E_t\pi_{H,t+1} + \frac{\alpha}{1 - \alpha} E_t\{\Delta q_{t+1}\}$$

Solving for $E_t\{\pi_{H,t+1}\}$ and substituting in equation (35)

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) - \left[ \frac{\alpha}{\sigma (1 - \alpha)} \right] E_t\{\Delta q_{t+1}\} - \left[ \frac{\alpha (2 - \alpha)(\gamma \sigma - 1)}{\sigma (1 - \alpha)} \right] E_t\{\Delta q_{t+1}\} - E_t\{\Delta g_{t+1}\}$$
\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(\bar{u}_t - E_t\{\pi_{t+1}\}) - \alpha \left[ \frac{1+(2-\alpha)(\gamma\sigma-1)}{\sigma(1-\alpha)} \right] E_t\{\Delta q_{t+1}\} - E_t\{\Delta g_{t+1}\} \quad (36) \]

### 2.7 SUPPLY SIDE EQUILIBRIUM

The New Keynesian Philips curve (NKPC hereafter) with real marginal cost cannot be estimated directly due to unavailability of data on Marginal cost in national income accounts. Generally, two methods are employed in literature to estimate NKPC by replacing marginal cost through an appropriate proxy variable. The two methods are Output Gap method and Unit real labour cost method. We proceed by establishing the relationship between marginal cost and economic activity through labour and goods market clearing conditions following (GM, 2005) and (CGG, 2002). The marginal cost function derived from labour market (equation (25)) can be transformed in real terms as below.

\[ mc_t = -\eta a_t + \eta(w_t - p_t) + (1 - \eta)(p_{O,t} - p_t) - (p_{H,t} - p_t) \quad (37) \]

Substituting intertemporal optimality condition (eq. 10) and equation (16) we get

\[ mc_t = -\eta a_t + \eta(\sigma c_t + \varphi n_t) + (1 - \eta)\bar{p}_{O,t} + \frac{\alpha}{1-\alpha} q_t \]

where, \( \bar{p}_{O,t} = p_{O,t} - p_t \) is the inflation adjusted price of oil. Using log-linearized version of cost minimization (eq. 22), aggregate production function (eq. 26), and risk sharing conditions (eq. 18), we get

\[ mc_t = -\psi_1 a_t + \psi_2 y_t^* + \psi_3 y_t + \psi_4 \bar{p}_{O,t} + \psi_5 q_t \quad (38) \]

The above equation shows negative relationship between marginal cost and productivity and positive relationship between domestic and foreign output and real price of oil,

where, \( \psi_1 = \frac{\eta(1+\varphi)}{1+\varphi(1-\eta)}, \psi_2 = \frac{\eta\sigma}{1+\varphi(1-\eta)}, \psi_3 = \frac{\eta\varphi}{1+\varphi(1-\eta)}, \psi_4 = \frac{(1+\varphi)(1-\eta)}{1+\varphi(1-\eta)}, \psi_5 = \frac{\eta}{1+\varphi(1-\eta)} + \frac{\alpha}{1-\alpha} \)

Let \( \sigma = \left[ \frac{\alpha\sigma(2-\alpha)+(1-\alpha)^2}{\sigma(1-\alpha)} \right] \), then (eq. 33) can be written as

\[ y_t = y_t^* + \sigma q_t + g_t \quad (33.A) \]
Solving (eq. 33.A) for $y_t^* = y_t - \omega q_t - g_t$ and substituting in (eq. 38) we get

$$mc_t = -\psi_1 a_t + (\psi_2 + \psi_3) y_t - \psi_2 g_t + (\psi_5 - \psi_2 \omega) q_t + \psi_4 \bar{p}_{0,t}$$

where, $\psi_6 = (\psi_2 + \psi_3)$ and if there is no price rigidity and all firms can adjust their price optimally in each period under flexible price setting, then there will be no markup differential and all the firms will charge equal markup.

$$\overline{mc}_t = -\mu$$

where, $\overline{mc}_t$ is the flexible price equilibrium constant marginal cost and $\mu = \log \left( \frac{\theta}{1-\theta} \right)$. If $\bar{y}_t$ shows the flexible level of output, then solving (eq. 39) for flexible price output;

$$\bar{y}_t = \frac{-\mu + \psi_1 a_t + \psi_2 g_t - (\frac{1}{1-\alpha} - \psi_2 \omega) q_t - \psi_4 \bar{p}_{0,t}}{\psi_6}$$

Forwarding one period ahead.

$$\bar{y}_{t+1} = \frac{-\mu + \psi_1 a_{t+1} + \psi_2 g_{t+1} - (\frac{1}{1-\alpha} - \psi_2 \omega) q_{t+1} - \psi_4 \bar{p}_{0,t+1}}{\psi_6}$$

where, $x_t = y_t - \bar{y}_t$ is the output gap, according to Output-Gap method, marginal cost is considered as cyclical in nature and varies directly with the gap between actual output and potential output. When actual output is greater than potential output, the competition for available factors of production will push their prices up and consequently the real marginal cost increases. Further, marginal cost is also influenced by the exchange rate and a rise in oil price in the short run for a country like Pakistan. Thus

$$\overline{mc}_t = \psi_8 x_t + \psi_9 q_t + \psi_{10} \bar{p}_{0,t}$$

### 2.7.1 The New-Keynesian Phillips Curve

After putting eq. (43) in equation (27), the NKPC can be expressed as below:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda \psi_8 x_t + \lambda \psi_9 q_t + \lambda \psi_{10} \bar{p}_{0,t} + \varepsilon_t^w$$

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2.7.2 The New-Keynesian IS Curve

Now combining (41), (42), (36), using the definition of output gap, the AR(1) productivity process and AR(1) oil price process the New Keynesian IS curve can be written as

\[ y_t - \bar{y}_t = E_t\{y_{t+1}\} - E_t\{\bar{y}_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) - \alpha \left[ \frac{1+(2-\alpha)\gamma(\sigma-1)}{\sigma(1-\alpha)} \right] E_t\{\Delta q_{t+1}\} - E_t\{\Delta g_{t+1}\} + E_t\{\bar{y}_{t+1}\} - \bar{y}_t \]

Given that,

\[ E_t\{\bar{y}_{t+1}\} - \bar{y}_t = -\frac{\psi_1}{\psi_6} (1 - \rho_a) a_t - \frac{\psi_1}{\psi_6} E_t\{\bar{\pi}_{o,t+1}\} + (\frac{\psi_2}{\psi_6} - 1) E_t\{\Delta g_{t+1}\} \]

Substituting for \( E_t\{\bar{y}_{t+1}\} - \bar{y}_t \) and rearranging we get

\[ x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) - \left[ \frac{1}{1-\alpha} \frac{\psi_2}{\psi_6} + \alpha \left( \frac{1+(2-\alpha)\gamma(\sigma-1)}{\sigma(1-\alpha)} \right) \right] E_t\{\Delta q_{t+1}\} - \frac{\psi_1}{\psi_6} E_t\{\bar{\pi}_{o,t+1}\} + \frac{\psi_1}{\psi_6} (\rho_a - 1) a_t + (\frac{\psi_2}{\psi_6} - 2) E_t\{\Delta g_{t+1}\} \]

As we have assumed productivity and government expenditures as exogenous shocks therefore, the above equation can be rewritten as follows.

\[ x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) - \left[ \frac{1}{1-\alpha} \frac{\psi_2}{\psi_6} + \alpha \left( \frac{1+(2-\alpha)\gamma(\sigma-1)}{\sigma(1-\alpha)} \right) \right] E_t\{\Delta q_{t+1}\} - \frac{\psi_1}{\psi_6} E_t\{\bar{\pi}_{o,t+1}\} + \varepsilon_t^{af} \]

2.8 OIL PRICE SETTING

We assume that the small open economy is net oil importer, and price taker in international oil market. Further we assume full exchange rate pass-through as observed in Pakistan since 2001. During this period oil prices are revised on monthly basis by Oil Company Advisory Committee (OCAC) and later on by Oil and Gas Regulatory Authority (OGRA). Thus, price of oil in domestic currency imported from country \( i \) and aggregate oil price imported from rest of the world can be expressed as:
\[ p_{O,t} = p_{O,t}^* + \nu_t \]
\[ \tilde{p}_{O,t} = \tilde{p}_{O,t}^* + q_t \]  
(47)

where, \( \nu_t \) is the log nominal exchange rate and \( q_t \) is the log effective real exchange rate. Moreover, as the domestic economy has no power in setting oil price hence the oil price variable is taken as exogenous following as AR(1) process.

\[ \tilde{p}_{O,t} = \rho_o \tilde{p}_{O,t-1} + \varepsilon_t^{po} \]  
(48)

where, \( \varepsilon_t^{po} \) is i.i.d., and \( \rho_o \in [0,1) \).

### 2.9 Monetary Policy Reaction Function

We assume that monetary policy is conducted according to forward looking Taylor rule with defined objectives such as stability regarding CPI Inflation and exchange rate along with rapid economic growth.

The welfare loss function in an efficient steady state under commitment is given as (CGG, 1999).

\[ L = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \phi_x x_t^2 + q_t^2) \]

This loss function is maximized subject to following constraints:

(a) \[ x_t = E_t \{ x_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \}) - \left[ \frac{1 - \alpha}{\psi_6} + \alpha \left( \frac{1 + (2 - \alpha)(\gamma \sigma - 1)}{\sigma(1 - \alpha)} \right) \right] E_t \{ \Delta q_{t+1} \} - \frac{\psi_1}{\psi_6} (1 - \rho_o) a_t - \frac{\psi_4}{\psi_6} E_t \{ \tilde{p}_{O, t+1} \} + \left( \frac{\psi_2}{\psi_6} - 1 \right) E_t \{ \Delta g_{t+1} \} \]

(b) \[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda k x_t + \varepsilon_t^w \]

(c) \[ E_t \{ \Delta q_{t+1} \} = (i_t - E_t \{ \pi_{t+1} \}) - (i_t^* - E_t \{ \pi_{t+1}^* \}) \]

Solving the above system yields the following monetary authority reaction function:

\[ i_t = \phi_\pi E_t \{ \pi_{t+1} \} + \phi_x x_t + \phi_q E_t \{ \Delta q_{t+1} \} + \varepsilon_t^r \]  
(49)
3. Conclusion

The present study has derived a proto-type DSGE model by augmenting the GM (2005) small open economy model to capture the structure and functioning of the Pakistani economy. Various sources of fluctuations such as the role of fiscal policy shock, monetary policy shock, oil price shock, cost push shock and risk premium shock have been internalized. Thus the model can be used to evaluate the response of key macroeconomic variables to these shocks.

This model is the starting point that can be extended in various directions by relaxing the underlying assumptions to make it tractable and closer to the real world economy. Furthermore, incorporating the labour market imperfections, introducing financial sector, international trade, and fiscal sectors would provide further strength to investigate the inter-linkages between these sectors. Welfare analysis associated with different policy stances is another important direction of research. As far as the estimation of these models and forecasting of key macroeconomic variables is concerned various alternatives are currently in vogue.\footnote{An in-depth discussion on extended DSGE model, estimation procedures, and policy analysis can be found in Fagiolo and Roventini (2017).}

APPENDIX-A

The representative household tries to minimize the cost of achieving the required level of composite consumption good $c_{it}$ with market price $p_{jt}$.

$$C_t = \left( \int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (A.1)$$

Maximize $c_{it}$

$$\left[ \int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Subject to

$$\int_0^1 P_{jt} c_{it} \, di = Z_t \quad (A.2)$$

$$\mathcal{L} = \left[ \int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \left( \int_0^1 p_{jt} c_{it} \, di - Z_t \right)$$

FOC yields;
\[ C_{it} : \frac{\varepsilon}{\varepsilon - 1} \left( \int_0^1 C_{it}^{\varepsilon - 1/\varepsilon} \, d\varepsilon \right)^{\varepsilon - 1} \frac{\varepsilon}{\varepsilon - 1} C_{it}^{\varepsilon - 1} - \lambda_t P_{it} = 0 \]

\[ \left( \int_0^1 C_{it}^{\varepsilon - 1/\varepsilon} \, d\varepsilon \right)^{1/\varepsilon - 1} C_{it}^{1/\varepsilon} - \lambda_t P_{it} \]

\[ = \left[ \left( \int_0^1 C_{it}^{\varepsilon - 1/\varepsilon} \, d\varepsilon \right)^{\varepsilon - 1} \right]^\frac{1}{\varepsilon} C_{it}^{1/\varepsilon} - \lambda_t P_{it} = 0 \]

As \( \int_0^1 C_{it}^{\varepsilon - 1/\varepsilon} \, d\varepsilon = C_t \) hence,

\[ C_t^{1/\varepsilon} C_{it}^{1/\varepsilon} = \lambda_t P_{it} \]

In case of two differentiated goods the equality must hold so; \( \left( \frac{C_{it}}{C_{jt}} \right)^{1/\varepsilon} = \frac{P_{it}}{P_{jt}} \) or

\[ C_{it} = C_{jt} \left( \frac{P_{it}}{P_{jt}} \right)^{-\varepsilon} \quad (A.3) \]

Insert (this) into the budget constraint:

\[ Z_t = \int_0^1 P_{it} C_{it} \, d\varepsilon \]

\[ = \int_0^1 P_{it} C_{jt} \left( \frac{P_{it}}{P_{jt}} \right)^{-\varepsilon} \, d\varepsilon \]

\[ = P_{jt}^{\varepsilon} C_{jt} \int_0^1 P_{it}^{1-\varepsilon} \, d\varepsilon \]

\[ C_{jt} = \frac{z_t P_{jt}^{\varepsilon}}{\int_0^1 P_{it}^{1-\varepsilon} \, d\varepsilon} \]

Insert the above result into (A.1)

\[ C_t = \left( \int_0^1 C_{it}^{\varepsilon - 1/\varepsilon} \, d\varepsilon \right)^{1/\varepsilon - 1} \]
\[ Z_t = \left( \int_0^1 P_{1t}^{1-\epsilon} \, di \right)^{1-\epsilon} \]

**Which represents the aggregate price index. The optimal consumption level can be find out by substituting (A.3) into the budget constraint.**

\[ Z_t = P_{1t} P_i \]
Solving for $C_{it}$

$$C_{it} = \frac{Z_t}{P_t} \left( \frac{P_{it}}{P_t} \right)^{-\epsilon}$$  \hspace{1cm} (A.5)

Insert into (A.1) and solving for $C_{it}$ to get the final demand function for good $i$:

$$C_t = \left[ \int_0^1 C_{it}^{\frac{\epsilon}{\epsilon-1}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$= \left( \int_0^1 \frac{Z_t}{P_t} \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} \frac{1}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$= \left( \int_0^1 \frac{P_{it}^{-\epsilon}}{Z_t P_t^{\epsilon-1}} \frac{1}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$= Z_t P_t^{\epsilon-1} \left[ \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{\frac{1}{\epsilon-1}} \right]^{-\epsilon}$$

$$= Z_t P_t^{\epsilon-1-\epsilon}$$

$$= Z_t P_t^{-1}$$

$$\Rightarrow \int_0^1 P_{it} C_{it} \, di = Z_t = P_t C_t$$

$$C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} C_t$$  \hspace{1cm} (A.6)

**APPENDIX-B**

The representative household maximizes his life time utility subject to a period by period budget constraint via langrangian.
\[
L = E_0 \sum_{t=0}^{\infty} \beta_t \left( \frac{C_t^{1-\sigma} N_t^{1+\varphi}}{1-\sigma} + \frac{(M_t)^{1-\nu}}{1-\nu} \right)
- \lambda_t \left( P_t C_t + M_t + E_t [F_{t,t+1} D_{t+1}] - D_t - W_t N_t - M_{t-1} - T_t \right)
\]

Taking first order partial derivatives with respect to consumption, money holdings, labor supply and dividend income we yield the following conditions.

\[ C_t: \quad \beta_t C_t^{1-\sigma} - \lambda_t P_t = 0 \quad (B.1) \]

\[ \frac{\beta_t C_t^{1-\sigma}}{P_t} = \lambda_t \]

Iterating (eq-B.1) one period forward we get:

\[ C_{t+1}: \quad \beta_t E_t C_{t+1}^{1-\sigma} - \beta E_t \lambda_{t+1} P_{t+1} = 0 \quad (B.2) \]

\[ M_t: \quad (M_t)^{-\nu} - \lambda_t + \beta E_t \lambda_{t+1} = 0 \quad (B.3) \]

\[ N_t: \quad - \beta_t N_t^{\varphi} + \lambda_t W_t = 0 \quad (B.4) \]

\[ \frac{\beta_t N_t^{\varphi}}{W_t} = \lambda_t \]

\[ D_{t+1}: \quad \lambda_t E_t \{ F_{t,t+1} \} = \beta \lambda_{t+1} \quad (B.5) \]

From (eq-B.5)

\[ R_t = \frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{1}{E_t \{ F_{t,t+1} \}} \]

As \( E_t \{ F_{t,t+1} \} = \frac{1}{R_t} \) or \( R_t = 1/E_t \{ F_{t,t+1} \} \)

Solving (eq-B.1) and (eq-B.4) we get the intertemporal optimality condition

\[ C_t^{\varphi} N_t^{\varphi} = \frac{W_t}{P_t} \quad (B.6) \]
From (eq-B.1), (eq-B.2) and (eq-B.5) we get the Euler equation showing the inter-temporal substitution in consumption

$$\beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1$$  \hspace{1cm} (B.7)

Money demand function is obtained by solving (eq-B.1) and (eq-B.3)

$$\frac{M_t}{P_t} = C_t^\sigma \left( \frac{1+i_t}{i_t} \right)^{1/\gamma}$$  \hspace{1cm} (B.8)

References


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