A Time Series Analysis of Aggregate Consumption Function for Pakistan

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School of Social Sciences and Humanities (S^3H)
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Abstract .................................................................................................................................................ii
Introduction..................................................................................................................................................1
The Data & Methodology .........................................................................................................................2
Methodology................................................................................................................................................2
  The DHSY Model ....................................................................................................................................2
  Random Walk Model ............................................................................................................................3
  Martingale Hypothesis ............................................................................................................................4
  Campbell and Mankiw Model ...............................................................................................................4
Results Discussion: ....................................................................................................................................5
  The DHSY Model (Error Correction Model) .........................................................................................5
    The DHSY’s Unrestricted Model.........................................................................................................7
      1. Price Homogeneity ......................................................................................................................7
      2. Unit Elasticity of Income ..........................................................................................................8
  Random Walk Model ..............................................................................................................................8
  The Martingale Hypothesis ....................................................................................................................9
  Campbell and Mankiw Model .............................................................................................................9
CONCLUSION ............................................................................................................................................10
References: ................................................................................................................................................11
Abstract

This study provides a time series analysis of aggregate consumption function for Pakistan by using a quarterly data from year 1973(1)-2010(4). The application of DHSY’s error correction and Hall’s random walk models are empirically tested. The data validate the application of both methodologies in estimating the aggregate consumption function. Hall’s Martingale hypothesis also holds which show that in Pakistan current consumption is a good predictor of future consumption. In context of Pakistan, 86% of the income has been consumed in the long-run while rest is the saving. Inflation is unanticipated but not accelerating. The data provide enough evidence to reject the price homogeneity hypothesis however; we are unable to reject the hypothesis of unit elasticity of income. The Mankiw and Campbell test concludes that there are 49% consumers who are backward looking while 51% follow permanent income hypothesis and are forward looking.

Key Words: Random Walk, Error Correction, Martingale, Price Homogeneity

JEL Classification: B22, B23, C12, C22
Introduction

Aggregate consumption determines aggregate savings which entail a portion of national income not consumed. Aggregate consumption and aggregate savings have powerful influence on an economy’s long term productivity capacity and capture the macroeconomic fluctuations and business cycles more appropriately. This attention reflects the belief that the structural forms of consumption functions and theories are a key to solve many business cycle issues.

Consumption theories have evolved over the decades and different consumption functions are formulated. After the great depression of 1930, consumption theories became focus of many economists and researchers. The most famous work is done by Keynes (1936) who developed consumption function in the form of Absolute Income Hypothesis (AIH). The consumption theories have radically changed after 1950’s by the emergence of Permanent Income Hypothesis (PIH) presented by Milton Friedman (1957) and Life Cycle Hypothesis (LCH) developed by Modigliani and Brumberg (1957). In 1989, Campbell and Deaton calculated the proportion of individual who follow Rule of Thumb (ROT) i.e., AIH and those who are forward looking and follow PIH.

After the plethora of work on these theories by many economists, Hall (1978) presented another theory of consumption which is known as Random Walk Model. A major portion of the empirical literature on aggregate consumption is focused on Hall’s demonstration that current consumption depends only on its own lag. Hall is followed by many researchers who have tested LCPI hypothesis under rational expectations such as Bilson (1980), Flavin (1981), Blinder and Deaton (1985) and Mankiw (1982). He also investigated that assuming under some assumptions the PIH implies that consumption must be a martingale process that is no variable other than current consumption helps in predicting future consumption. Haug (1991) proposed that empirical rejections of the LCPI model are due to the time aggregation bias. His empirical results support Hall’s hypothesis that consumption is random walk. The findings in Hayashi (1985) are in line with Hall and Mishkin (1982).

In 1978, the Davidson, Hendry, Srba, and Yeo, (DHSY) noticed that no consensus have emerged about the short-run dynamic interaction between the disposable income and consumer’s expenditure in the United Kingdom despite the superficial similarity of the published models. In
contention to explain the diversity of the published estimates. DHSY incorporated the dynamic econometric models to estimate consumption function. DHSY’s model was a revolution in the traditional consumption theories as it provided a more sophisticated technique to estimate consumption by incorporating all the previous consumption functions. All these theories and econometric models have been empirically evaluated by many economists in context of developing and developed economies. The results vary from country to country.

This study investigates the application of DHSY’s dynamic model and Hall’s random walk model in context of Pakistan by using quarterly aggregate consumption data from year 1973(1)-2010(4). We also investigate the ratio of consumers who follow ROT and those who follow PIH through Campbell and Mankiw (1989) test. Further more, we tested the hypothesis of (i) unit elasticity, (ii) price homogeneity, and (iii) martingale process in context of Pakistan.

The Data & Methodology

The data used in this study comprises of the following variables:

\[ \ln C_t = c_t \quad : \quad \text{Aggregate Consumption Expenditure constant at 1999-2000 prices} \]

\[ \ln Y_t = y_t \quad : \quad \text{Gross Domestic Product at Factor Cost constant at 1999-2000 prices} \]

\[ P_t \quad : \quad \text{Inflation as Implicit function of Consumption} \]

The data is collected on quarterly basis from 1973 to 2010 and acquired from a report published by State Bank of Pakistan on quarterization of national income accounts (Hanif, Iqbal, & Malik, 2013). The variables estimated are in million Rupees.

Methodology

The DHSY Model

DHSY showed that the Error Correction Model (ECM) would yield a robust short-run (dynamic) relationship between Aggregate Consumption Expenditures and Personal Disposable Income. DHSY considered the steady state theory and presented a model which would generate a long run unitary income elasticity of consumption.

DHSY use quarterly seasonally unadjusted data in their empirical analysis. The model to be considered here is:
\[ \Delta_4 c_t = \alpha_1 \Delta_4 y_t + \alpha_2 \Delta_1 \Delta_4 y_t + \alpha_3 \ln(C/y)_{t-4} + \alpha_4 \Delta_4 P_t + \alpha_5 \Delta_4 P_t + \epsilon_t \quad \text{(1)} \]

the symbols \( \Delta \) denote difference operators.

Under the permanent income hypothesis, if \( \alpha_1 \) is positive then some of the increase in income is treated as an increase in permanent income. If \( \alpha_2 \) is positive then if the consumer considers this extra rise to be transitory income; the consumer will reduce consumption from the level implied by treating the whole income increase as permanent. The alternative argument would hold if \( \alpha_2 \) is negative. Thus, on economic theory grounds \( \alpha_2 \) may be expected to be negative.

The restriction of dropping the intercept was considered by DHSY because they retain an ECM term of the form \( \ln(C/y)_{t-4} \).

For static equilibrium, the equation solves as:

\[ \alpha_3 (c_{t-4} - y_{t-4}) = 0 \]

and if \( \alpha_3 \neq 0 \) then \( c_t = y_t \) which implies a long run unitary income elasticity of consumption.

Inflation is often unanticipated with consumers interpreting the price rises of goods they typically purchase as relative price increases rather than as a general rise in the aggregate price level which may be because inflation is accelerating. This response of consumers to unanticipated inflation viewed in this way would be to reduce their real consumption leading to what Deaton termed “involuntary saving through unanticipated inflation” (Deaton, 1977). Hence despite consumer expenditure and income variables being measured in real terms, DHSY did also find a role for inflation effects. According to Deaton’s argument if inflation is unanticipated, \( \alpha_4 \) will be negative and if inflation is both accelerating and unanticipated then \( \alpha_5 \) will also be negative.

To test the hypothesis of price homogeneity (\( \sum \tilde{\beta}_i = 0 \)) and unit elasticity of income (\( \sum \tilde{\alpha}_i \approx 1 - \sum \tilde{\gamma}_i \)) following model is used.

\[ C_t = \sum_{i=0}^{5} (\alpha_i Y_{t-i} + \beta_i P_{t-i}) + \sum_{i=1}^{5} (\gamma_i C_{t-i}) + \epsilon_t \quad \text{(2)} \]

**Random Walk Model**

Hall (1978) combined the life cycle and permanent income models with the rational expectations to conclude that consumption follows the random walk that is changes in consumption
over time are unpredictable. The final equation of RWM with drift can be written in terms of a regression model as:

\[ c_t = \beta_1 c_{t-1} + \epsilon_t \quad (3) \]

The Random Walk hypothesis implies that there is no need for other variables for forecasting because all the information is already there in \( C_{t-1} \) decision and adding other variables have no predictive power and they will be statistically insignificant (Hall, 1978).

**Martingale Hypothesis**

Another implication of Hall’s model is that consumption follows a martingale. It implies that change in consumption should be uncorrelated with unanticipated changes in income i.e.

\[ E_{t-1}(\Delta c_t) = 0 \]

To test this implication following model is used which is in fact a special case of Hall’s 1978 test of the Martingale model for consumption.

\[ \Delta c_t = \alpha + \sum_{i=1}^{n} \beta_i \Delta y_{t-i} + \epsilon_t \quad (4) \]

\[ H_0: \beta_1 = \beta_2 = \ldots = \beta_n = 0 \Rightarrow c_t \text{ is Martingale.} \]

**Campbell and Mankiw Model**

Campbell and Mankiw in 1990 separated the proportion of forward looking consumers from backward looking consumers. They assumed that a proportion (\( \lambda \)) follow the “Rule of Thumb (ROT)” and consume their current income (AIH) while proportion (1-\( \lambda \)) individuals are forward looking and satisfy the Permanent Income Hypothesis (PIH).

For Rule of Thumb consumers:

\[ \Delta c_{t}^{rot} = \Delta y_{t}^{rot} \]

For Permanent Income Hypothesis followers:
\[ \Delta c_t^p = \varepsilon_t \]

where \( E_{t-1} \varepsilon_t = 0 \). Since \( \Delta c_t = \Delta c_t^{rot} + \Delta c_t^p \) and \( \Delta y_t^{rot} = \lambda \Delta y_t \), we can have

\[ \Delta c_t = \lambda \Delta y_t + \varepsilon_t \quad ----- (5) \]

The error term is an innovation to consumption plus other possible errors. The model (5) cannot be estimated directly by OLS because \( y_t \) may be correlated to \( \varepsilon_t \). The standard instrumental technique may be applied but we will be using the following alternative method.

Assume,

\[ \Delta y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + u_t \quad ------ (6) \]

If we estimate this model by OLS, we get \( \hat{\alpha} \) which converges to \( \alpha \) in large samples. Substituting eq. (6) in eq. (5), we get:

\[ \Delta c_t = \lambda (\alpha_0 + \alpha_1 \Delta y_{t-1} + u_t) + \varepsilon_t \]

\[ \Delta c_t = \beta_0 + \beta_1 \Delta y_{t-1} + \eta_t \quad ----- (7) \]

where, \( \beta_1 = \alpha_1 \lambda \Rightarrow \lambda = \beta_1 / \alpha_1 \) and \( \eta_t = \lambda u_t + \varepsilon_t \) has expectation equal to zero and it uncorrelated with the lagged variables.

The instrumental method (IV) method breaks down if \( \alpha \) is in the vicinity of zero (due to weak instrument) particularly when we divide out by \( \hat{\alpha} \) rather than \( \alpha \), our results can be very noisy.

**Results Discussion:**

**The DHSY Model (Error Correction Model)**

The estimation result of final model of DHSY i.e., eq.(1) is given below:

\[ \Delta_4 \hat{C}_t = 0.37 \Delta_4 \hat{Y}_t + 0.79 \Delta_1 \Delta_4 \hat{Y}_t - 0.099 ln \left( \frac{C}{Y} \right)_{t-4} - 0.046 \Delta_4 \hat{P}_t + 0.079 \Delta \Delta_4 \hat{P}_t + 0.57 \Delta_4 \hat{C}_{t-1} \]

T-statistic: \( (4.09) \quad (7.02) \quad (-2.91) \quad (-1.98) \quad (2.06) \quad (8.51) \)
All the variables are in log form and are seasonally differenced. The diagnostic tests show no autocorrelation; no heteroscedasticity and no model misspecification. According to the feedback theory, consumers plan to spend their income in each year same as they spend in the previous year modified by a proportion of their annual change in income (i.e. $a_1 = 0.37$) and by whether the change in income is itself is increasing or decreasing (i.e. $a_2 = 0.79$). These together determine a short-run consumption decision which is altered by ($\alpha_3 = -0.099$) ensuring coherence with the long-run outcome $C_t = KY_t$. The impact elasticity is $1.158 \ (0.37 + 0.79)$ falling to $0.37 \ (37\%)$ after one quarter. If the change in income is assumed to be a permanent, initially people would consume more than the change in income (they entail to purchase durable goods or other things they have long planned but couldn’t because of income constraint) but the consumption will go down after one quarter to $37\%$. However, the long-run MPC is $0.86 \ [0.37/ \ (1-0.57)]$ which corroborates with the findings in Khan (1993).

The sign of Error Correction Term (ECM) is negative i.e., $\alpha_3 < 0$ which shows that long run APC is inversely related to the growth rate with which consumption and income are growing. Also, $\alpha_3 \neq 0$ which confirms that $C_t = Y_t$ and implies a long run unitary income elasticity of consumption. This rate of adjustment is $10\%$ which is very slow. If speed of adjustment is slow then APC is expected to decline (Fig. 1).

Under Deaton’s argument, the negative sign of inflation coefficient shows that inflation is unanticipated and positive sign of rate of inflation shows that it is not accelerating (Deaton, 1977). The response of consumers to unanticipated inflation would be to reduce real consumption leading to what Deaton has termed “Involuntary saving through unanticipated inflation”.

The Fig. 2 below shows that there is a good agreement between model forecasts and realizations. Both the Forecast Chi-square and Chow tests imply the parameter stability between the samples and post sample periods as all the projections are within the $\pm 2$ SD bounds. The RMSE is $0.0128$ showing low forecast error.
The DHSY’s Unrestricted Model

The estimation results for eq. (2) are given below:

\[
\hat{\epsilon}_t = 1.128 \hat{\rho}_t - 0.84 \hat{\rho}_{t-1} + 0.01 \hat{\rho}_{t-2} - 0.037 \hat{\rho}_{t-3} - 0.32 \hat{\rho}_{t-4} + 0.247 \hat{\rho}_{t-5} - 0.029 \hat{\rho}_t - 0.133 \hat{\rho}_{t-1} + 0.06 \hat{\rho}_{t-2} + 0.05 \hat{\rho}_{t-3} - 0.032 \hat{\rho}_{t-4} + 0.07 \hat{\rho}_{t-5} + 0.64 \hat{\epsilon}_{t-1} - 0.02 \hat{\epsilon}_{t-2} + 0.07 \hat{\epsilon}_{t-3} + 0.26 \hat{\epsilon}_{t-4} - 0.139 \hat{\epsilon}_{t-5}.
\]

In this model all the variables are on levels and in log form. The diagnostic tests show no evidence of autocorrelation, heteroscedasticity and model misspecification. The unrestricted form of the DHSY’s error correction is estimated to test the following hypothesis:

1. **Price Homogeneity**

\[ H_0 : \sum \hat{\beta}_1 = 0 \Rightarrow \text{Prices are homogeneous} \]

\[ H_1 : \sum \hat{\beta}_1 \neq 0 \Rightarrow \text{Prices are heterogeneous} \]
2. **Unit Elasticity of Income**

\[ H_0 : \sum \tilde{\alpha}_t + \sum \tilde{\gamma}_t = 1 \]

\[ H_1 : \sum \tilde{\alpha}_t + \sum \tilde{\gamma}_t \neq 1 \]

The parameter restriction tests reveal that the hypothesis of homogeneous prices does not hold in case of Pakistan (\( \text{Chi}^2 = 10.63 \ [0.001] \)). However, we fail to reject the unit elasticity of income (\( \text{Chi}^2 = 2.26 \ [0.132] \)). It also confirms that \( C_t = Y_t \) which implies Long-run unitary income elasticity of consumption.

**Random Walk Model**

The estimation of final equation of Random Walk Model (RMW) is:

\[ \hat{C}_t = 1.00093 \hat{C}_{t-1} \]

SE: \( (0.00094) \)

\[ H_0 : \beta_1 = 1 : \text{Chi}^2(1) = 0.99005 \ [0.3197] \]

The data do not provide enough evidence to reject the null hypothesis. It proves the validity of Hall’s Random walk model in case of Pakistan. The results and test statistics are significant.

![Eight Quarters forecast (RWM)](image)

Fig. 3  Eight Quarters forecast (RWM)

The forecast shows that all the values of projection are within the 95% forecast bounds. The parameter constancy test (Chow F(8,142) = 1.4673 [0.1743]) imply that the null hypothesis cannot be rejected i.e., No parameter value changes between the sample and post sample periods. The forecast accuracy is validated by the chi-square test (Forecast \( \text{Chi}^2(8) = 11.738 \ [0.1633] \)).
The Martingale Hypothesis

Regressing consumption on five lags of income by OLS (eq. 4), we get following estimated equation:

\[ \Delta C_t = 0.024 + 0.19\Delta Y_{t-1} + 0.26\Delta Y_{t-2} + 0.10\Delta Y_{t-3} - 0.196\Delta Y_{t-4} + 0.126\Delta Y_{t-5} \]

(2.23)  (1.16)  (1.53)  (0.6)  (-1.18)  (0.8)

\[ H_0: \beta_1 = \beta_2 = \ldots = \beta_5 = 0 \implies C_t \text{ is Martingale} \]

\[ H_1: \text{At least one } \beta_i \neq 0 \implies C_t \text{ is not Martingale.} \]

The F-test does not allow to reject the null hypothesis (\( F(5,137) = 1.8027 \ [0.1163] \)). This implies that consumption follows a martingale process i.e. no variable other than current consumption can help predicting future consumption.

Campbell and Mankiw Model

Campbell and Mankiw test help us to determine the proportion of individuals who follow the rule of thumb (ROT). Equations 6 & 7 are estimated and solved for the parameter \( \lambda \), the proportion of those who follow ROT.

\[ \Delta_4 Y_t = 0.03 + 0.39\Delta_4 Y_{t-1} \]

(7.07)  (5.09)

\[ \Delta_4 C_t = 0.03 + 0.19\Delta_4 Y_{t-1} \]

(4.70)  (1.31)

\[ \lambda = \frac{\beta_1}{\alpha_1} = \frac{0.19}{0.39} = 0.49 \]

This implies that 49% individuals follow ROT and 51% follow PIH. Those consumers who follow rule of thumb i.e., Keynes absolute income hypothesis believe that current consumption depends on current income and consumption changes with the change in income. However, the PIH followers respond to changes in their permanent income. Hence, the temporary changes in their income (associated to business cycle) have little effect on their consumption. They utilize their saving to smooth their consumption pattern.
CONCLUSION

Aggregate Consumption being an important part of National Income Accounts, has been intensively researched in macroeconomics. Aggregate consumption and aggregate savings have powerful influence on an economy’s long term productivity capacity and capture the macroeconomic fluctuations and business cycles more appropriately. This attention reflects the belief that the structural forms of consumption functions and theories are a key to solve many business cycle issues.

This study empirically proves the validity of Hall’s Random Walk Model and DHSY’s Error Correction Model in context of Pakistan by using quarterly data of consumption from year 1973(1)-2010(4). Both models provide stable relationship between consumption and income but DHSY’s model has a slight edge over the RWM in terms of forecast analysis. Hall's Martingale hypothesis also holds which show that in Pakistan current consumption is a good predictor of future consumption.

In context of Pakistan, 86% of the income has been consumed in the long-run while rest is the saving. Inflation is unanticipated but not accelerating. The data provide enough evidence to reject the price homogeneity hypothesis however; we are unable to reject the hypothesis of unit elasticity of income. The Mankiw and Campbell test concludes that there are 49% consumers who still follow ROT and are backward looking while 51% follow PIH and are forward looking.
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